

# A note on large $N$ scalar QCD<sub>2</sub>

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We review the features of the bound state equation in large  $N$  scalar QCD in two dimensions, the 't Hooft model, and compute the discrete hadron mass spectrum in this theory. We make the Ansatz that the scalar fields of this model represent spin zero diquarks and we estimate the minimum allowed mass for the first radial excitation of the lowest diquark-antidiquark scalar meson. The discussion is extended to the case of spin one diquarks.

**Introduction.** In a recent paper on scalar meson dynamics [1] it is shown how a satisfactory explanation of light scalar meson decays can be reached by assuming a dominant diquark-antidiquark structure,  $q\bar{q}$ , for the lightest particles. The diquark  $q$  is a spin zero antitriplet color state. In first approximation the nonet formed by  $f_0(980)$ ,  $a_0(980)$ ,  $\kappa(900)$ ,  $\sigma(500)$  is interpreted as the lowest  $q\bar{q}$  multiplet. On the other hand, the decuplet of scalar mesons with masses above 1 GeV, formed by  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1710)$ ,  $a_0(1450)$ ,  $K_0(1430)$ , and likely containing the lowest glueball, is interpreted in [1] as the lowest  $q\bar{q}$  scalar multiplet (see also [2]). The underlying hypothesis that the latter multiplet is not a radial excitation of the former has never been proven.

In this note we provide evidence in support of this hypothesis by estimating the mass of the first radial excitation of the lowest sub-GeV  $q\bar{q}$  scalar meson.

We perform this calculation in large- $N$  QCD in  $(1+1)$ -dimensions. This is a planar, linearly confining theory which admits a Bethe-Salpeter equation describing the discrete spectrum of  $q\bar{q}$  bound states [3]. In this theory no orbital angular momentum excitations are possible since no rotation operator can be introduced: the discrete spectrum describes radial excitations.

Our Ansatz is that the scalar fields of large  $N$  scalar chromodynamics in  $(1+1)$ -dimensions (sQCD<sub>2</sub>) can be thought as diquark fields. The corresponding Bethe-Salpeter equation of this theory should then yield the spectrum of tetraquarks  $q\bar{q}$  particles.

The bound state equations in spinor and scalar chromodynamics are respectively (see e.g. [4]):

$$\mu_{(\pi)}^2 \phi(x) = \frac{M_f^2}{x(1-x)} \phi(x) - \int_0^1 dy \frac{1}{(y-x)^2} \phi(y) \quad (1)$$

$$\mu_{(\sigma)}^2 \phi(x) = \frac{M_s^2 - 1}{x(1-x)} \phi(x) - \int_0^1 dy \frac{1}{(y-x)^2} \frac{(x+y)(2-x-y)}{4x(1-x)} \phi(y) \quad (2)$$

where the integrals are in the sense of the Cauchy principal value and  $\phi(0) = \phi(1) = 0$ . Here  $\mu_{(\pi)}^2$  and  $\mu_{(\sigma)}^2$  are the mass squared eigenvalues of standard  $q\bar{q}$  mesons, call them pions, and tetraquark  $q\bar{q}$  mesons, sigmas. The parameters  $M_f$  and  $M_s$  represent the masses of the quark  $q$  and of the scalar quark  $q$  respectively. In this notation all the masses are adimensional parameters since we are rescaling them by the couplings  $g^2$ : in two-dimensional scalar and spinor chromodynamics the couplings have dimension of mass.

We cannot expect that the mass spectra in (s)QCD<sub>2</sub> reproduce numerically the physical values of the masses of real pions and sigmas but we can assume that the regularities in the spectra of this kind of hadron-string models resemble those in the physical ones. To begin, we require that the ratio between the ground states  $\mu_{(\sigma)}/\mu_{(\pi)}$  corresponds to the ratio between the lowest lying tetraquark object, the  $\sigma(500)$ , and the lowest standard  $q\bar{q}$  meson, the pion. We find that this is allowed by a family of values of the parameters  $(M_f, M_s)$ .

The first observed meson with the same quantum numbers of the pion is the  $\pi(1300)$  [5] which is therefore a good candidate for the first radial pion excitation. Considering that all bound states are alternately even or odd under parity [6] and that the ground state for a  $q\bar{q}$  particle has positive parity, the bound state corresponding to  $\pi(1300)$  is the second excited state in the spectrum  $\mu_{(\pi)}$ . Then a simple calculation allows to estimate the mass of the first expected physical radial excitation of the  $\sigma(500)$  meson.

It turns out that, spanning the set of allowed masses  $(M_f, M_s)$ , the minimum value for the first radial excitation of the  $\sigma(500)$  tetraquark is at about 2600 MeV. This would predict an extremely broad state hardly identifiable experimentally. Moreover this result underscores that there is no possibility

that the above 1 GeV scalar mesons in the range  $1350 \div 1700$  MeV could be excitations of the below 1 GeV ones. This strengthens the hypothesis that in the latter mass region there is space only for the lowest  $q\bar{q}$  multiplet (and likely a glueball).

In the last section we extend the discussion to the case of heavy-light diquarks introduced to describe the  $X, Y, Z$  particles found by Belle and BaBar. In particular we will focus on the charged  $Z(4430)$  particle recently observed by Belle. As this is a spin one resonance, spin one diquarks have to be considered. We find that the bound state equation for scalar chromodynamics can be exploited also for a discussion of heavy-light axial diquarks.

We start by illustrating the method used to solve Eqs. (1,2) and the numerical results obtained.

**Large  $N$  scalar  $QCD_2$ .** We will briefly review scalar chromodynamics in (1+1) dimensions and outline the derivation of Eq. (2). A thorough discussion of Eq. (1) in spinor chromodynamics is found in [3]. Omitting color indices, the scalar chromodynamics Lagrangian density in light-cone coordinates

$$x^\pm = \frac{x^0 \pm x^1}{\sqrt{2}} \quad (3)$$

reads:

$$\mathcal{L} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - m^2 \varphi^\dagger \varphi + \frac{1}{2} (\partial_- A_+) (\partial^+ A_-) - \frac{g}{\sqrt{N}} A_+ (\varphi^\dagger \partial_- \varphi - (\partial_- \varphi^\dagger) \varphi) \quad (4)$$

where the  $\mu$  index is  $\mu = +, -$ , the gauge fields are in the adjoint representation of  $SU(N)$  whereas the scalar fields are in the fundamental one and the light-cone gauge  $A^+ = A_- = 0$  has been imposed. In this gauge no self-couplings of the gauge fields or seagull terms exist. The theory is asymptotically free and linearly confining. To make the self energy smooth at large  $N$ , we introduce  $1/\sqrt{N}$  at each vertex.

The emergence of a linearly confining potential can be observed by writing the equation of motion for the field  $A_+$ :

$$\partial_-^2 A_+ = -2 \frac{g}{\sqrt{N}} j_- \quad (5)$$

which admits the following solution (use  $\partial_- |x^-| = \text{sign}(x^-)$  and  $\partial_- \text{sign}(x^-) = 2\delta(x^-)$ ):

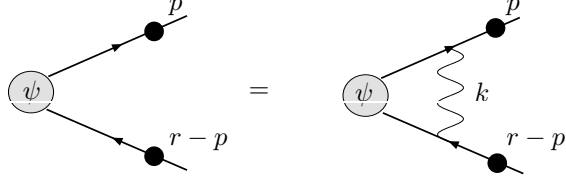
$$A_+ = -\frac{g}{\sqrt{N}} \int dy_+ |x_+ - y_+| j_- \quad (6)$$

in absence of background fields. Plugging Eq. (6) into Eq. (4) it turns out that there is a linear potential between charges. By dimensional analysis,  $g$  has dimensions of energy.

We will consider this theory in the 't Hooft limit: large  $N$  and  $g$  held fixed. As  $N = 3$  the color transformation property of a diquark (the scalar quark  $\varphi_\alpha$ ) and of an antiquark are the same: diquarks are  $qq$  states with attraction in the color  $\bar{\mathbf{3}}$  channel. Extending the color group to  $SU(N)$ , with  $N > 3$ , this diquark-antiquark correspondence is lost. Here we treat the fields  $\varphi_\alpha$  as the fields of building block, pointlike, diquarks. As for color transformations, this is appropriate for  $N = 3$ , whereas for larger  $N$  it is an extrapolation. Alternatively one could consider the Corrigan-Ramond large  $N$  limit [7]. In the latter case quarks and 'larks' are introduced, transforming as the  $\mathbf{N}$  and  $\mathbf{N}(\mathbf{N} - 1)/2$  representations of  $SU(N)$  respectively. Larks  $\ell$  are represented by antisymmetric tensors  $\ell_{\alpha\beta} = -\ell_{\beta\alpha}$  coinciding with antiquarks if  $N = 3$ . A theory of only larks is equivalent to QCD. In the CR large  $N$  limit, a baryon is represented by  $qq\ell$  whereas there are no color singlets made up of three quarks in the standard large  $N$ .

In the large  $N$  limit only planar diagrams are relevant and quark loops are suppressed. Hence gluon lines are impassable barriers as there are no gluon-gluon interactions and any gluon line crossing would violate planarity.

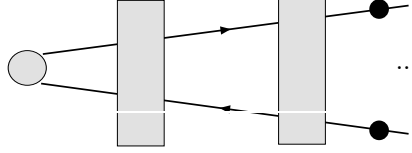
Eq. (2) is derived from the Bethe-Salpeter equation expressed by the following diagrammatic relation:



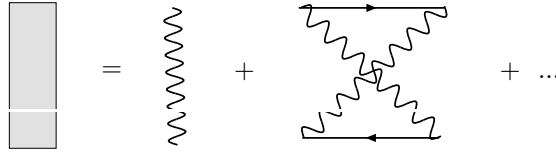
where the disks on the external legs indicate the dressed propagators. The shaded blobs represent the matrix element of the time ordered product of two scalar quark fields between the vacuum  $|0\rangle$  and the meson state  $|B\rangle$  [3]. We will denote by  $\psi$  the Fourier transform of this matrix element. The Bethe-Salpeter equation is the Dyson-Schwinger equation for a 4-point Green function  $G$  where a bound state  $|B\rangle$  occurs as a mass pole (the residue at the pole giving the bound state wave function) in  $G$ :

$$G \rightarrow \frac{|B\rangle\langle B|}{q^2 - \mu^2 + i\epsilon}. \quad (7)$$

The simplicity of the previous diagrammatic equation is a consequence of the large  $N$  limit. Let us represent the Dyson-Schwinger series as:



In the large  $N$  limit, the only non-vanishing contribution from the amputated functions represented by the shaded boxes is the first one:



Once the gluon propagator is found to be (see [3]):

$$D_{\mu\nu} = i\delta_{\mu+}\delta_{\nu+}\frac{\mathbb{P}}{k_-^2} \quad (8)$$

where  $\mathbb{P}$  indicates the Cauchy principal value, it is not difficult to compute the quark dressed propagators  $D(p)$ . This can be seen by observing that:

$$\oint dk_- \frac{e^{ik_-x_+}}{k_-^2} = |x_+| \quad (9)$$

One obtains (cfr. [4]):

$$D(p) = \frac{i}{2p_- \left( p_+ - \frac{g^2}{2\pi} \left( \frac{\text{sign}(p_-)}{\lambda} - \frac{1}{p_-} \right) + \frac{i\epsilon - M^2}{2p_-} \right)} \quad (10)$$

where  $\lambda$  is an infrared cutoff. To derive the latter expression it is convenient to observe that:

$$\int dp_+ \frac{p_-}{2p_+p_- - A + i\epsilon} = -i\frac{\pi}{2}\text{sign}(p_-) \quad (11)$$

as can be proved by using the Plemelj identity:

$$\int dx \frac{f(x)}{x - x_0 \mp i\epsilon} = \oint dx \frac{f(x)}{(x - x_0)} \pm i\pi f(x_0) \quad (12)$$

and that  $\mathbb{P}/x = 0$ .

According to Eq. (10), in the limit  $\lambda \rightarrow 0$  the quarks have infinite self-energy, hence are removed from the spectrum. Anyway the dependency on  $\lambda$  is canceled in the Bethe-Salpeter equation which eventually defines the eigenvalue equation for scalar quark bound states; in other words we have a discrete spectrum of bound states but no free quarks. The diagrammatic equation given above writes as:

$$\psi(p, r) = g^2 D(p) D(p - r) \oint \frac{d^2k}{4\pi^2} \frac{i}{k_-^2} \psi(p + k, r) (2p + k)_- (2p - 2r + k)_- \quad (13)$$

Note the derivative couplings of scalar chromodynamics at the vertices of the gluon propagator. Observe also that the kernel of the principal value integral depends on minus momenta only: this is clear from Eq. (6) where only  $j_-$  currents are coupled to the linear potential, or equivalently from the expression of the gluon propagator. We can therefore integrate both sides of Eq. (13) over  $p_+$ . A straightforward residue calculation gives:

$$\int dp_+ \frac{1}{(p_+ - A)(p_+ - r_+ - B)} = -\frac{2\pi i \theta(p_-) \theta(r_- - p_-)}{A - B - r_+} \quad (14)$$

where we required a negative imaginary part for the pole  $A$  and a positive one for the pole  $B$ . If both poles were on the same half-plane the residue integration would yield zero closing the contour in the other half-plane. We close the contour of integration in the lower-half plane (clockwise sign). Defining:

$$\phi(p_-, r) \equiv \int dp_+ \psi(p_+, p_-, r) \quad (15)$$

one can write Eq. (13) as:

$$\phi(p_-, r) = -\frac{g^2}{2\pi} \frac{\theta(p_-) \theta(r_- - p_-)}{r_+ - (A - B)} \int_0^{r_-} \frac{dk_-}{(k_- - p_-)^2} \phi(k_-, r) \frac{(p_- + k_-)}{2p_-} \frac{(p_- + k_- - 2r_-)}{2(p_- - r_-)} \quad (16)$$

The latter integral is infrared divergent when  $p_- \sim k_-$ . We can expand it as:

$$\begin{aligned} \int_0^{r_-} \frac{dk_-}{(k_- - p_-)^2} \phi &= \oint_0^{r_-} \frac{dk_-}{(k_- - p_-)^2} \phi - \phi(p_-, r) \lim_{\lambda \rightarrow 0} \int_{p_- - \lambda}^{p_- + \lambda} \frac{dk_-}{(k_- - p_-)^2} = \\ &= \oint_0^{r_-} \frac{dk_-}{(k_- - p_-)^2} \phi + \frac{2}{\lambda} \phi(p_-, r) \end{aligned} \quad (17)$$

This expression allows to cancel the  $1/\lambda$  infrared divergence in Eq. (16). Consider in fact that:

$$A - B = \frac{g^2}{\pi} \frac{1}{\lambda} + \frac{M^2 - g^2/\pi}{2p_-} - \frac{M^2 - g^2/\pi}{2(p - r)_-}. \quad (18)$$

Eq. (2) is obtained by defining  $r_+ = \mu^2/(2r_-)$ ,  $xr_- = p_-$  and  $yr_- = k_-$  and rescaling all the square masses in units of  $g^2/\pi$ . Observe that the  $\theta$ -functions in Eq. (14) define an interval outside which  $\phi = 0$ . This interval is  $p_- \in [0, r_-]$  or  $x \in [0, 1]$ . In particular we have  $\phi(0) = \phi(1) = 0$ .

The integral in Eq. (2) gives its main contribution if  $y \simeq x$ , where the kernels of Eq. (1) and (2) are the same. As the highest part of the spectrum is concerned, one can neglect  $M_{f,s}$  and the eigenfunctions are approximated by  $\phi(x) \simeq \sin \omega x = \sin n\pi x$ , with  $n$  integer  $n > 1$ . This approximation respects the condition  $\phi(0) = \phi(1) = 0$ . For periodic functions  $\phi(x)$  we have:

$$\oint_0^1 dy \frac{\exp(i\omega x)}{(y - x)^2} \simeq \oint_{-\infty}^{\infty} dy \frac{\exp(i\omega x)}{(y - x)^2} = i\omega \oint_{-\infty}^{\infty} dy \frac{\exp(i\omega x)}{(y - x)} = -\pi|\omega| \exp(i\omega x) \quad (19)$$

as can be seen easily by applying (12). This result means that the eigenvalues in the highest part of the spectrum are  $\mu_n^2 \simeq n\pi^2$ , i.e., there is *no continuum* in the spectrum [3]. The alternating parity for these  $\phi_n(x)$  states is evident.

In what follows we will focus on the lower part of the spectrum, and we will solve Eqs. (1) and (2) with a numerical approach.

**Discrete Spectra.** Equations (1,2) are integral equations with singular kernels and a prescription, the Cauchy principal value, on how to treat the singularity. In this section we will indicate the steps to put them in a form amenable to numerical computation. We follow a procedure which has been applied in [8] (and reference therein) to solve Eq. (1). The main point is to write:

$$\phi(\theta) = \sum_{m=0}^N a_m \sin m\theta \quad (20)$$

and set:

$$x = \frac{1 + \cos \theta}{2} \quad (21)$$

$$y = \frac{1 + \cos \theta'}{2} \quad (22)$$

Integrating by parts in Eq. (1) to lower the singularity of the kernel, and exploiting the conditions  $\phi(0) = \phi(1) = 0$ , one obtains an integral of the form:

$$\oint_0^\pi d\theta' \frac{\cos m\theta'}{\cos \theta' - \cos \theta} \quad (23)$$

which can be solved using the following relation between Chebyshev polynomials [9]

$$\oint_{-1}^1 dy \frac{T_n(y)}{(y-x)\sqrt{1-y^2}} = \pi U_{n-1}(x), \quad (24)$$

where  $x \in [-1, 1]$ . Indeed this is done by observing that  $T_n(\cos \theta) = \cos n\theta$  whereas  $U_n(\cos \theta) = \sin(n+1)\theta / \sin \theta$ . The next step is to discretize the angle  $\theta$  according to the prescription [8]:

$$\theta \rightarrow \theta_k = \frac{k\pi}{N+1} \quad (25)$$

and to use the following orthogonality relation:

$$\sum_{k=1}^N \sin k\theta_m \sin k\theta_n \equiv \sum_{k=1}^N \sin \theta_{km} \sin \theta_{kn} = \frac{N+1}{2} \delta_{mn} \quad (26)$$

to transform Eq. (1) into an eigenvalue equation of the form:

$$\sum_{m=1}^N \mathcal{O}_{nm}^{(\pi)} a_m = \mu_{(\pi)}^2 a_n \quad (27)$$

where we have [8]:

$$\mathcal{O}_{nm}^{(\pi)} = \frac{4}{N+1} \sum_{k=1}^N \frac{\sin \theta_{kn} \sin \theta_{km}}{\sin \theta_k} \left( \frac{2M_f^2}{\sin \theta_k} + m\pi \right). \quad (28)$$

The operator  $\mathcal{O}_{nm}^{(\pi)}$  can be diagonalized and a set of discrete eigenvalues  $\mu_{(\pi)}$  can be found. We aim here to find an operator  $\mathcal{O}_{nm}^{(\sigma)}$  and an eigenvalue equation form for Eq. (2) similar to that in (27). The procedure to follow is basically the same as the one outlined above, just requiring some more algebra. An helpful relation to use to obtain  $\mathcal{O}_{nm}^{(\sigma)}$  is provided by the following identity relating Chebyshev polynomials, similar to that given in Eq. (24) [17]:

$$\oint_{-1}^1 dy \frac{\sqrt{1-y^2} U_{n-1}(y)}{(y-x)} = -\pi T_n(x) \quad (29)$$

The result is:

$$\begin{aligned} \mathcal{O}_{nm}^{(\sigma)} = & \frac{2}{N+1} \sum_{k=1}^N \left[ \left( \frac{4(M_s^2 - 1)}{\sin^2 \theta_k} + \frac{7}{16} m\pi \frac{1}{\sin \theta_k} \right) \sin \theta_{nk} \sin \theta_{mk} - \frac{\pi}{2} \cos \theta_k \sin \theta_{nk} \cos \theta_{mk} + \right. \\ & - \frac{m\pi}{8} \cot \theta_k \sin \theta_{nk} (\sin \theta_{mk} \cos \theta_k + \sin \theta_{(m+1)k} + \sin \theta_{(m-1)k}) + \\ & \left. - \frac{m\pi}{32} \frac{\sin \theta_{nk}}{\sin \theta_k} (\sin \theta_{(m+2)k} + \sin \theta_{(m-2)k}) \right]. \end{aligned} \quad (30)$$

In the derivation of  $\mathcal{O}_{nm}^{(\sigma)}$  the trigonometric Werner identities have been used. Thus we have to pick up a minimum value of  $M_s$  to avoid tachyons.

The operator obtained allows to write also Eq. (2) in a form amenable to numerical computation:

$$\sum_{m=1}^N \mathcal{O}_{nm}^{(\sigma)} a_m = \mu_{(\sigma)}^2 a_n \quad (31)$$

and can be diagonalized to obtain the spectrum of eigenvalues  $\mu_{(\sigma)}$ .

**Numerical results.** In Figure 1 we show a set of values  $(M_s, M_f)$  which give a ratio  $\mu_{(\sigma)}/\mu_{(\pi)}$  compatible with the experimental one, taking the mass value of the  $\sigma$  from [1]. These values can be

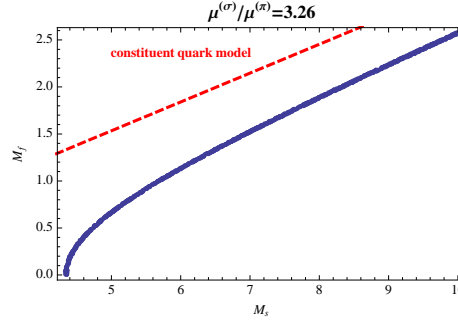


FIG. 1: The curve represents the  $(M_s, M_f)$  values which give the ratio  $\mu_{(\sigma)}/\mu_{(\pi)}$  of the lowest eigenvalues equal to the experimental one. We check that for large values of  $(M_f, M_s)$  the two equations (1) and (2) are the same and the results converge with what expected from naive constituent quark model. The numerical analysis is performed by setting the  $N$  parameter in Eq. (20) to  $N = 100$ . The results are stable under variation of  $N$ .

used to determine the gap between the ground levels and the first excited levels, having the desired parity, in scalar and spinor chromodynamics. The first radially excited  $0^-$  state is expected to be the second excited state in the  $q\bar{q}$  spectrum, the ground state being  $0^-$ . Define the following ratio:

$$\lambda = \left( \frac{\mu_{(\pi)}^{[2]}}{\mu_{(\pi)}^{[0]}} \right) / \left( \frac{\mu_{(\sigma)}^{[2]}}{\mu_{(\sigma)}^{[0]}} \right) \quad (32)$$

where with the superscripts 0, 1, 2, ... we label the levels in the spectrum: 0 is the ground state. The predicted value for the first positive parity radial excitation of a system of two scalar quarks (a diquark-antidiquark state in our language) is then  $m_{(\sigma)}^{[2]}$  which can be obtained by:

$$m_{(\sigma)}^{[2]} = \frac{m_{\pi'}}{m_{\pi}} \frac{1}{\lambda} \quad (33)$$

using the mass  $m_{\pi'} = 1300$  MeV for the first radial excitation of the pion and  $m_{\pi} = 135$  MeV,  $m_{\sigma} = 470$  MeV. With the values  $(M_s, M_f)$  shown in Fig. 1 it turns out that  $m_{(\sigma)}^{[2]} \gtrsim 2590$  MeV indicating that a radial excitation of the lowest lying tetraquark state would be likely a very broad object. Also there is no harm to find an exotic radial excitation in the range of masses where the

super-GeV scalar nonet is located. Similarly we can also estimate the lower bound for the first radial  $0^-$  excitation of a  $q\bar{q}$  particle, finding  $m_{(\sigma)}^{[1]} \gtrsim 3060$  MeV. This can be done by assuming that the first  $q\bar{q}$  scalar radial excitation is the  $f_0(1370)$ . The given minimum value occurs *e.g.* at the point  $(M_f, M_s) \sim (0.1, 4.3)$  of the parameter space spanned by the curve in Fig. 1.

**Heavy-light diquarks.** Diquarks have been used in the literature also to discuss a number of recently discovered charmonium-like resonances, known as  $X, Y, Z$  particles, with phenomenological properties conflicting with those of standard charmonium states. The  $X(3872)$  found by Belle [10], was recognized from the start to be a very anomalous charmonium state, despite its discovery decay mode into  $J/\psi\rho$  that could have seemed the footprint of a higher charmonium. In [11] the  $X(3872)$  is interpreted as a  $q\bar{q}$  state with  $q = [cq]$ ,  $q$  being a light quark  $q = u, d$ . The diquark model predicts two charged almost degenerate  $X$  particles and two neutral ones, namely  $[cu][\bar{c}\bar{u}]$  and  $[cd][\bar{c}\bar{d}]$ . As discussed in [12] there is strong evidence by BaBar and Belle that a second neutral state, the  $X(3876)$ , exists! As for now, there are no conclusive indications for  $X^\pm$ . On the other hand Belle has found three charged resonances decaying into charmonium+charged  $\pi$ , namely  $Z_1(4051), Z_2(4248), Z(4430)$ . These are produced in  $B \rightarrow KZ_1, Z_2$  or  $Z$ . One can find a brief account on this in [15].

For example, the  $Z(4430)$ , observed in  $Z \rightarrow \psi(2S)\pi^+$  [13], if confirmed, should necessarily be a multiquark object. In [14] this state is interpreted as a  $[cu]_1[\bar{c}\bar{d}]_0$  radial excitation of a  $1^{+-}$  particle predicted in [11] and decaying into  $\psi(1S)\pi^+$ .

Interestingly the difference in mass between the radially excited tetraquark  $Z^+$  and its fundamental state predicted in [11] is very close to  $M_{\psi(2S)} - M_{\psi(1S)}$ .

The notation  $[cu]_1[\bar{c}\bar{d}]_0$  means that we assume one of the two diquarks to have spin one. Light diquarks with spin one are thought to be less stable than spin zero ones, but, as a heavy-quark is introduced in the diquark, spin-spin chromomagnetic interactions are suppressed by the heavy mass and one expects to treat heavy-light diquarks with spin zero and spin one on the same footing. For a phenomenological investigation on higher light tetraquarks see [16].

In this section we want to investigate the system  $q_1\bar{q} + q\bar{q}_1$  using the methods developed above. We assume that the spin one diquark can be described by a colored (axial) field  $\mathcal{A}_i$  in the  $\mathbf{\bar{3}}$ -color representation. The gluon field in the adjoint is  $A_j^i$ , with  $A_i^i = 0$  and  $A_j^i = -A_i^{*j}$  if the minimal coupling is  $\partial_\mu \rightarrow \partial_\mu + gA_\mu$ . We define  $\mathcal{F}_{\mu\nu}^i = (\partial_\mu A_\nu^i - \partial_\nu A_\mu^i)$  for the axial diquark.

The vertex  $\mathcal{A}\mathcal{A}\mathcal{A}$  can be extracted by:

$$\frac{g}{\sqrt{N}}[\mathcal{A}_i^\mu \mathcal{A}^{\nu j}(F_{\mu\nu})_j^i + \mathcal{A}_i^\mu \mathcal{F}_{\mu\nu}^j (A^\nu)_j^i + \mathcal{F}_{\mu\nu i} \mathcal{A}^{\mu j} (A^\nu)_j^i] \quad (34)$$

or in terms of Feynman rules:

$$= \frac{g}{\sqrt{N}}[g_{\alpha\nu}(k-p+r)_\mu + g_{\alpha\mu}(-p+r-2k)_\nu + g_{\mu\nu}(2p-2r+k)_\alpha]$$

where  $\alpha, \mu, \nu = +, -$ . The light cone gauge fixes  $\alpha = -$  and  $A = (\epsilon, -\epsilon)$ . If the incoming  $\mathcal{A}$  is  $\mathcal{A} = (0, \epsilon)$  then the outgoing one is  $\mathcal{A} = (\epsilon, 0)$ , *i.e.*, at the vertices we have  $\mathcal{A}_+\mathcal{A}_-$  or vice-versa. We sum over these two alternatives. Taking  $\alpha = -, \nu = -, \mu = +$ , from (34) we get  $(-p+r-2k)_- + (2p-2r+k)_- = (p-r-k)_-$  whereas taking  $\alpha = +, \nu = +, \mu = -$  we have  $(k-p+r)_- + (2p-2r+k)_- = (p-r+2k)_-$ . The sum of the two contributions involves  $(2p-2r+k)_-$  as in (13). The same considerations are applied to the calculation of the self-energy  $(\mathcal{A}\mathcal{A}\mathcal{A})$  which therefore proceeds in the same way as in the scalar chromodynamics case.

This allows to use the same Bethe-Salpeter equation given in (2) to study the case of charmed diquark-antidiquark states of the form  $[cu]_1[\bar{c}\bar{d}]_0 + [cu]_0[\bar{c}\bar{d}]_1$ .

In [11], using chromomagnetic interaction Hamiltonians, the following mass values of  $1^{+-}$ ,  $[cq][\bar{c}\bar{q}]$  states, were found:  $Z(3754)$  and  $Z(3882)$ . In [14] the hypothesis is made that the observed  $Z^+(4430)$  could be a radial excitation of the  $Z(3882)$ . Here we assume that these three states,  $Z(3754), Z(3882), Z(4430)$  correspond to  $\mu^{[0]}, \mu^{[2]}, \mu^{[4]}$  of Eq. (2). If we require that  $\mu^{[2]}/\mu^{[0]} = 1.08 = M_{Z(3882)}/M_{Z(3754)}$  we see that the minimum allowed value of the parameter  $M_s$  (we could call it  $M_a$  in this case making reference to axial diquarks) is  $M_s \sim 10.43$ , quite higher than what found before,

consistently with the idea that we should describe spectra of charmed (*i.e.* heavier) particles. With the latter value of  $M_s$  one finds  $\mu^{[4]}/\mu^{[2]} \sim 1.20$  which requires that the value of the mass of  $Z(4430)$ , as found in this model, is indeed 4658 MeV. This value is rather stable if one increases the parameter  $M_s$  up to  $M_s \sim 20$ . The next radial excitation is predicted at 6055 MeV.

**Conclusions.** In this short note we assume that scalar chromodynamics in (1+1)-dimensions and with a large number of colors could be used to estimate the mass of the first radial excitation of a diquark-antidiquark meson. We applied a numerical method to solve the Bethe-Salpeter equations and compute the bound state discrete spectrum of this confining theory. The possible masses of the spinor and scalar quarks are found by imposing that the ratio of the ground state eigenvalues of the spinor and scalar Bethe-Salpeter equations, Eqs. (1) and (2) respectively, is equal to the ratio of the physical masses  $m_\pi/m_\sigma$ . Furthermore, with these masses, we were able to extract a minimum value for the first radial excitation of the ground state diquark-antidiquark spectrum. We extend our discussion to the heavy-light diquark sector finding that the  $Z^+(4430)$  observed by Belle could correspond to the second radial excitation of the spectrum  $Z(3754)$ ,  $Z(3882)$ ,  $Z(4658)$ ... where the lower states were predicted in [11].

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